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# Is Stretching and Folding Feature of Chaotic Trajectories Useful in Adaptive Local Projection?

Sir,

Chaotic behavior is a feature associated with complex and interacted systems. Many natural and unnatural systems in various branches of science (such as biology, economics, etc.) exhibit chaotic behavior and the study of chaotic systems and signals has progressed in the recent decades.[1] Chaotic time series have a significant role in identification of their generating systems. It has been claimed that in medical science many signals like brain signals (both microscopic and macroscopic ones),[2,3] cardiac signals (e.g., ECG and HRV[1,4]), respiratory sounds,[5] etc. have chaotic properties. Owing to the effect of measurement instruments and the environment, all experimental data are mixed with noise to some extent. This fact is often undesirable. In other words, noise is an unwanted part of data.[1,6]

Different methods for removing noise from chaotic signals have been introduced. One of the best methods is the Local Projection approach,[1] The local projection approach projects the chaotic data in a neighborhood onto a certain hyperplane. Selection of neighborhood radius, which is mainly determined by the way of experience or trial-and-error methods, has a direct impact on its performance. There are a few works on choosing the neighborhood radius adaptively.[7-9]

A noticeable feature associated with chaotic trajectories is their stretching and folding.[10] An idea would be to make use of the large radius in trajectory points where the behavior is in the form of stretching. In folding points where the trajectory changes its direction abruptly, a small neighborhood radius can be used. Therefore a criterion would be required to measure the level of stretching and folding. Considering the geometrical features of the trajectory, it can be observed that folding occurs wherever higher curvature exists in the trajectory. We know that the curvature of a curve like $\vec{R}(t)=(r_1(t),r_2(t),\ldots,r_m(t))$ is calculated by the following equation:[11]

$$\kappa(t)=\frac{\left|\dot{\vec{R}}\times\ddot{\vec{R}}\right|}{\left|\dot{\vec{R}}\right|^3}. \quad (1)$$

Even though for clean signals this equation provides the curvature accurately, numerical methods for calculation of the derivative in noisy signals are problematic; especially, the presence of the term $\left|\dot{\vec{R}}\right|^3$ in the denominator is troublesome when $\left|\dot{\vec{R}}\right|$ approaches zero. In order to surmount these problems, a more well-behaved term can be used which operates without these concerns. Therefore, the curvature at each point is considered as the angle between the vectors connecting a point to its two neighboring points in the trajectory. For example consider the famous Lorenz System:[10]

$$\begin{aligned}\dot{X}&=p(Y-X)\\ \dot{Y}&=-XZ+rX-Y\\ \dot{Z}&=XY-bZ.\end{aligned} \quad (2)$$

In the phase space, we have a three-dimensional trajectory in the form of $R(t)=(x(t),y(t),z(t))$. Consider a specific trajectory point at $t=t^*$, then the supposed angle can be computed as the angle between $\overrightarrow{R(t^*+\Delta t)-R(t^*)}$ and $\overrightarrow{R(t^*)-R(t^*-\Delta t)}$ where $\Delta t$ is the sampling time interval.

We believe that the use of this method can improve the local projection approach efficiency. In addition, this idea (using Stretching and folding feature and the way it should be measured) could also be used in other areas of chaotic signal processing.

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