Original Article

Online Handwritten Signature Verification and Recognition Based on Dual-Tree Complex Wavelet Packet Transform

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Abstract

Background: With the increasing advancement of technology, it is necessary to develop more accurate, convenient, and cost-effective security systems. Handwriting signature, as one of the most popular and applicable biometrics, is widely used to register ownership in banking systems, including checks, as well as in administrative and financial applications in everyday life, all over the world. Automatic signature verification and recognition systems, especially in the case of online signatures, are potentially the most powerful and publicly accepted means for personal authentication. **Methods:** In this article, a novel procedure for online signature verification and recognition has been presented based on Dual-Tree Complex Wavelet Packet Transform (DT-CWPT). **Results:** In the presented method, three-level decomposition of DT-CWPT has been computed for three time signals of dynamic information including horizontal and vertical positions in addition to the pressure signal. Then, in order to make feature vector corresponding to each signature, log energy entropy measures have been computed for each subband of DT-CWPT decomposition. Finally, to classify the query signature, three classifiers including k-nearest neighbor, support vector machine, and Kolmogorov–Smirnov test have been examined. Experiments have been conducted using three benchmark datasets: SVC2004, MCYT-100, as two Latin online signature datasets, and NDSD as a Persian signature dataset. **Conclusion:** Obtained favorable experimental results, in comparison with literature, confirm the effectiveness of the presented method in both online signature verification and recognition objects.

Keywords: *Dual-tree complex wavelet packet transform, Kolmogorov–Smirnov test, log energy entropy measure, online handwritten signature verification, signature recognition*

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Introduction

Conventionally, authentication of a person was conducted in two types, including knowledge-based or token-based methods. In case of knowledge-based authentication system, the authentication is conducted based on something the user knows; the answer of the user to the secrete question(s) such as personal identification number or password.[1] However, token-based authentication systems work based on something the user has, such as a driver's license or identity document card.[1] However, traditionally authentication systems suffer from several treats including forgotten or stolen. These issues are addressed using biometrics. Authentication of a person using physical or behavioral characteristics is known as biometric. In the case of physical biometrics, authentication has been conducted using direct measurements of a part of human body, such as fingerprint,[2] face,[3] and iris.[4] On the other hand, behavioral biometrics use the information of an action performed by the user such as voice,[5] gait,[6] and signature.[7] Biometric-based authentication systems use many different aspects of human physiology, chemistry, or behavior. However, selection of a suitable biometric depends on several factors including:

- Robustness; does not have substantial changes over time
- Distinctively; has a great variation over different subjects
- Availability; all intended people have this characteristic
- Accessibility; easy to collect
- Acceptability; intended people agree to be taken from them.[8]

Handwritten signature is one of the most valuable biometric traits, which has mostly been used for verification purposes in

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everyday life.[9] Financial and administrative institutions use handwritten signatures as legal means of verifying an individual's identity, all over the world.[9] In addition, handwritten signature has an important advantage of noninvasive and nonthreatening process by the majority of the users.[10] It should be noted that signatures written by different people are naturally different which is known as interclass variability. Furthermore, due to the impact of physical and emotional conditions of the person on its signatures, handwritten signatures of one person are not the same and this variability is known as intraclass variability.[10] Notably, in addition to high intraclass variability, some other disadvantages are included in the case of handwritten signatures: forgeries, higher error rates than other biometrics, and large temporal variation.

A biometric system can be operated in two modes, namely recognition (identification) or verification. The aim of a verification system is the veracity of the person's claimed identity. However, an identification system tries to recognize the identity of the user.[8] A signature verification system deals with three kinds of forgeries: random, simple, and skilled forgeries.[11] In the case of random forgery, the forger without any information about the author's name and his/her signature reproduces a random signature. If the forger knows the author's name but does not access to any signature sample, the reproduced signature is known as a simple forgery. In the case of skilled forgery, a forger has signature samples and tries to reproduce them. Therefore, skilled forgeries are much more similar to the genuine signatures than random and simple forgeries. The focus of this paper, in the case of verification, is on the verification of skilled forgery samples.

It should be noted that training in an automatic signature verification system may be writer-independent (WI) or writer-dependent (WD).[12] In the first case, WI, training is conducted based on a large population of signature samples related to all persons in the dataset, whereas in the case of WD, training is done based on the signature samples of each person, separately.[12] Although WD approach achieves good results, for each user added to the system, a classifier must be conducted again which increases the complexity and cost of the system.[13] To reduce the complexity, WI approach attracts more researchers in recent years.[13,14] In this article, we consider WI approach and a novel procedure for online handwritten signature verification and recognition is presented.

Several reviews of the state-of-the-art on signature identification and signature verification have been recently proposed.[15-17] Input data of the signature-based authentication systems are in two types: offline (static) and online (dynamic). Offline handwritten signature-based authentication systems receive only the images of signatures gathered by a camera or a scanner. However, an online signature-based authentication system receives dynamic (temporal) information of the signing process in addition to the signature images which are gathered by a digitizing tablet or a pen-sensitive computer. Dynamic information includes time signals of horizontal position, $x(t)$, vertical position, $y(t)$, velocity, $v(t)$, pressure, $p(t)$, pen azimuth, $z\ (t)$, and altitude, $l\ (t)$.[10]

There are two main categories for feature extraction, i.e., information extracted from online signatures: global and functional. The aim of a global feature extraction method is to derive feature vectors of the same length and compare them.[18] Global feature extraction method divides into two categories: In the first category, features are extracted from the totally of the signature such as average pressure, average velocity, pen tip, total signing duration, and signature height,[18,19] while second category is dedicated to the features extracted by applying a transformation on the signature such as discrete wavelet transform (DWT) and discrete cosine transform.[20,21] On the other hand, functional features are dedicated to the features extracted by considering time sequences describing the signing process and calculating the distances between them.[18] Functional features are in two types; in the first type, a set of reference samples related to each subject has been saved as reference set. Then, classifying is done with the comparison of the input signature with the reference set using some methods such as Dynamic Time Warping (DTW). In the second type, a model is trained using the saved signature, and then, the test signature is classified using the trained model.[18]

An interesting field of research in engineering, mathematics, and bioinformatics is multiscale modeling. Multiscale modeling deals with problems with multiscale nature, i.e., having important features at multiple scales of time and/or space.[22] Notably, signals represented in time domain have important properties in the frequency domain.[23] Analysis of a time signal for its frequency content is effectively done using Fourier transform, wavelets, and other x-let transforms such as curvelet and contourlet.[23,24] Here, a novel signature verification/recognition method using DT-CWPT decomposition to obtain the frequency content of dynamic information of online signatures, has been presented.

The rest of this paper is organized as follows: a brief review on DT-CWPT with a literature review are presented in Subjects and Methods section. Details of the presented online signature verification method with the utilized datasets are explained in the Proposed online signature verification method section. Experimental results of signature verification and the comparison with literature are reported in the Experimental protocol and performance evaluation section. The presented online signature recognition method with the experimental results and comparison with literature are discussed in the Proposed online signature recognition method section. Finally, we conclude our work and outline some future work directions in the Conclusion and Future Works section.

Subjects and Methods

This section is dedicated to a brief review on basic concept used in the presented method, i.e., DT-CWPT, with a literature review on the related signature verification methods.

A brief review on dual-tree complex wavelet packet transform

Fourier transform is one of the most popular tools used in the field of multiscale modeling. In the Fourier transform, infinitely oscillating sinusoidal basis functions are used to represent the input signal. Some of the limitations of Fourier transform have been modified using wavelet transform.[20] Indeed, oscillating sinusoidal basis functions used in the Fourier transform are replaced with wavelets as basic functions which are locally oscillating in the DWT.[20] Wavelets are stretched and shifted versions of a real-valued bandpass wavelet □(*t*). To form orthonormal basis for the space of one-dimensional real-valued continuous-time signals, wavelets are combined with shifted versions of a real-valued low-pass scaling function □(*t*). Therefore, any finite energy signal *x*(t) can be decomposed in terms of wavelets and scaling function as follows:

$$x(t)=\sum_{n=-\infty}^{\infty}c(n)\varphi(t-n)+\sum_{j=0}^{\infty}\sum_{n=-\infty}^{\infty}d(j,n)2^{j/2}\Psi(2^{j}t-n) \tag{1}$$

Where □(.) and □(.) are computed using low-pass and high-pass filters, $h_0(n)$ and $h_1(n)$, respectively, as follows:

$$\Psi(t)=\sqrt{2}\sum_{n}h_0(n)\varphi(2t-n) \tag{2}$$

$$\varphi(t)=\sqrt{2}\sum_{n}h_1(n)\varphi(2t-n) \tag{3}$$

Further, the scaling coefficients, *c(n)*, and wavelet coefficients, *d(j, n)*, are computed using the inner product, as follows:

$$c(n)=\int_{-\infty}^{\infty}x(t)\varphi(t-n)\,dt \tag{4}$$

$$d(j,n)=2^{j/2}\int_{-\infty}^{\infty}x(t)\Psi(2^{j}t-n)\,dt \tag{5}$$

In DWT, only low frequency band is decomposed at each level of decomposition. The procedure of a two-level decomposition of DWT has been shown in Figure 1a. DWT has some advantages including good compression, perfect reconstruction, no redundancy, and very low computation.[25] However, DWT suffers from four deficiencies, including oscillations, shift variance, aliasing, and lack of directionality.[26]

An octave-band analysis of the frequency domain is provided by DWT which might not be optimal for a given signal. Finding an optimal representation is possible with discrete wavelet packet transform (DWPT).[27,28] In DWT, only low frequency band is decomposed at each level, but in DWPT, decomposition is conducted for both low and high frequency bands. It should be noted that DWPT is also shift-varying and suffers from lack of directionality in two-dimensions (2D) and higher dimensions, like DWT. These deficiencies are approximately addressed by dual-tree complex wavelet transform (DT-CWT).[28]

The structure of DT-CWT is composed of two DWTs; coefficients of the first and second DWTs are combined as real and imaginary parts of DT-CWT. Suppose that the first and second wavelets are denoted by □(*t*) and □'(*t*), respectively. These wavelets are defined similar to Eq. 2 and Eq. 3 using $\{h_0(n), h_1(n)\}$ and $\{h_0'(n), h_1'(n)\}$, for □(*t*) and □'(*t*), respectively. Figure 1b shows the structure of two-level decomposition of DT-CWT. It should be noted that Hilbert transform of a function *U*(t), is computed as follows:

$$H(U)(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{U(\tau)}{t-\tau}d\tau \tag{6}$$

To improve the shiftability of DWT, in DT-CWT, the second wavelet, □'(*t*), is the Hilbert transform of the first wavelet, □(*t*), i.e. □' = *H*{□(t)}. Advantages of DT-CWT include low computation, limited redundancy, perfect reconstruction, good directional selectivity, good shift invariance, and analyticity in 1D.[25]

As the DWPT extending the DWT, DT-CWPT extends the DT-CWT.[28] DT-CWPT is obtained by iterating two perfect reconstruction filter banks on the low-pass and high-pass outputs.[28] The design of these filters is such that the response of each branch of the second wavelet packet filter bank is the discrete Hilbert transform of the corresponding branch of the first wavelet packet filter bank. Therefore, each subband of the DT-CWPT will be approximately analytic.[28] This is true when the filters used in the second wavelet packet of DT-CWPT are the same with the filters used in the first wavelet packet of it. Notably, similar to DT-CWT, approximately shift-invariance and good directional selectivity are provided with DT-CWPT. Figure 1c shows the structure of two-level decomposition of DT-CWPT. As shown in Figure 1c, for k-level decomposition of DWT, DT-CWT, and DT-CWPT of ID signal, the number of output subbands is k + 1, 2(k + 1), and 2^{k+1}, respectively.

Literature review

Traditional wavelets and extensions of wavelets, such as DWPT and DT-CWT, have been widely used in the structure of the presented signature-based authentication systems, until today. In the following, some of the presented online signature verification methods based on wavelet or similar transforms have been briefly reviewed. An online signature verification system based on DWT features and neural network classification has been presented in the study by Maged and Fahmy.[21] Three-level decomposition of DWPT has been implemented on time signals, including horizontal

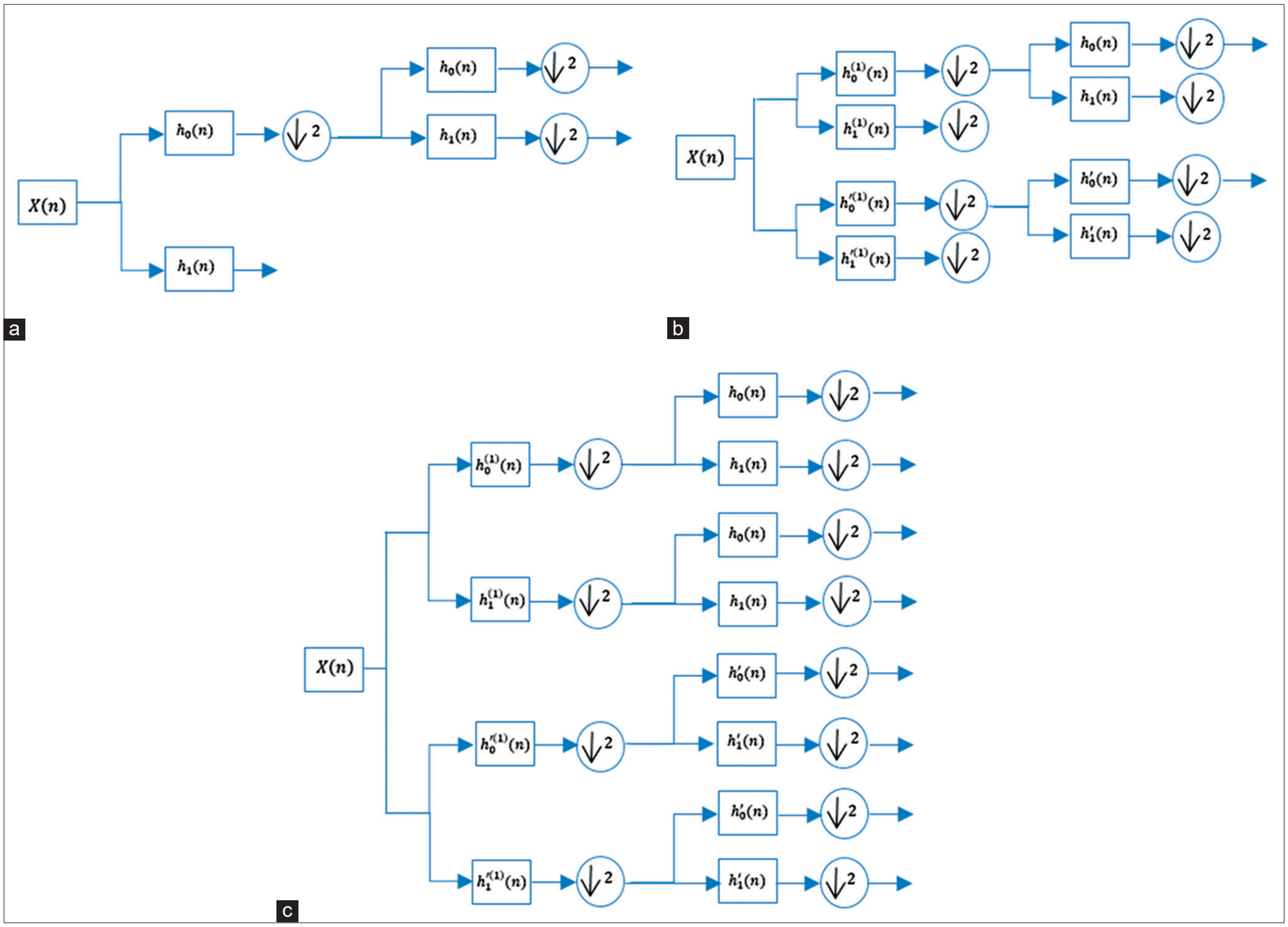

Figure 1: The structure of a two-level decomposition of a discrete wavelet transform (a) and dual-tree complex wavelet transform (b) and the structure of the first wavelet packet of a two-level decomposition of a dual-tree complex wavelet packet transform (c)

and vertical position, pressure, pen azimuth, and pen altitude, in the study by Wang *et al*.[29] Then, different combinations of these decompositions have been considered as extracted features. Finally, decision about genuine or forgery of the query signature has been made using the Euclidean distance. Continuous wavelet transform has been used to obtain the frequency information of the speed signals in.[30] The grayscale spectrograms created by wavelet transforms have been used to train support vector machine (SVM) network. In the study by Chang *et al*.,[31] five time signals including horizontal and vertical position, pressure, pen azimuth, and altitude have been considered for each signature. Then, different wavelets such as Haar, Daubechies, Symlet, and Coiflet have been used to decompose the time signal up to five-level decomposition. Final decision about the query signature has been conducted using a threshold on the computed distance measures. An improved wavelet-based online signature verification scheme has been presented in the study by Nilchiyan and Yusof.[32] In this work, five time signals including horizontal and vertical position, pressure, pen azimuth, and altitude have been discussed, and three-level decomposition of them using Haar and Daubechies wavelets have been computed for feature extraction. Classification has been done using the multi-perceptron neural network with one intermediary layer.

It should be noted that, in comparison with signature verification, there are fewer works conducted for online signature recognition, until today. To the best of our knowledge, there is no online signature recognition work based on wavelet or other similar transforms. However, there are few other procedures for online signature recognition, which have been briefly reviewed in the following. An online signature slant identification was presented in the study by Shamsuddin and Mohamed.[33] Horizontal and vertical positions of the signing process have been considered and filtered them. Then, the angle and degree of the signature have been computed and the signature has been classified into its slant category using a slant algorithm. This work presented an accuracy of 80% on a private dataset. In the study by Mohamed *et al*.,[34] a baseline extraction algorithm has been used for online signature recognition based on vector rules. Direction, slant, baseline, pressure, speed, and numbers of pen ups and downs have been used as the main features. An algorithm to extract baseline from signature has been used and obtained the accuracy of 90% on their created dataset. An online signature recognition based on some global features such as standard deviation and maximum of the velocity and the acceleration in both x and y directions has been presented in the study by Al-Mayyan *et al*.[35] A rough set classifier has been used and the accuracy of 100% for recognition rate has been obtained on its own created dataset composed of 2160 signatures from 108 subjects. Concepts of graph theory have been used for online signature identification in the study byFotak *et al*.[36] A fast classification using graph norm and comparison between each signature graph concepts value with the saved values in the dataset has been conducted to obtain an identification accuracy of 94.25% on their own dataset.

However, to the best of our knowledge, DT-CWPT has not been used in the structure of online signature recognition and verification systems, until today. In this paper, a writer-independent online handwritten signature recognition and verification system is presented based on DT-CWPT decomposition of time signals containing dynamic information of each signature.

Proposed online signature verification method

In this section, the novel presented method for online handwritten signature verification based on DT-CWPT has been described in detail. The block diagram of the presented signature verification method is shown in Figure 2.

Online handwritten signature datasets

The performance of the presented online signature verification method has been evaluated using three benchmark available datasets, including SVC2004[37] and Ministerio De Ciencia Y Tecnologia (MCYT)-100,[38] as two benchmark online handwritten Latin signature datasets, and Noshirvani Dynamic Signature Dataset (NDSD),[39] as an online handwritten Persian signature dataset. The statistics of these datasets are displayed in Table 1.

SVC2004 (https://www.cse.ust.hk/svc2004/) is an online handwritten Latin signature dataset established for the First International Signature Verification Competition (SVC2004).[37] WACOM INTUOS tablet was used for saving the dynamic information of the signing process. SVC2004 dataset provided two different signature databases, namely Task 1 and Task 2. The main SVC2004 dataset includes 100 contributors: 40 persons included in Task 1 and other 60 persons included in Task 2 signature dataset. Task 2, which has been considered in this study, contains 40 signers with 20 genuine signatures and 20 skilled forgeries per signer. Genuine signatures were signed in 2 weeks and skilled forgeries were signed with

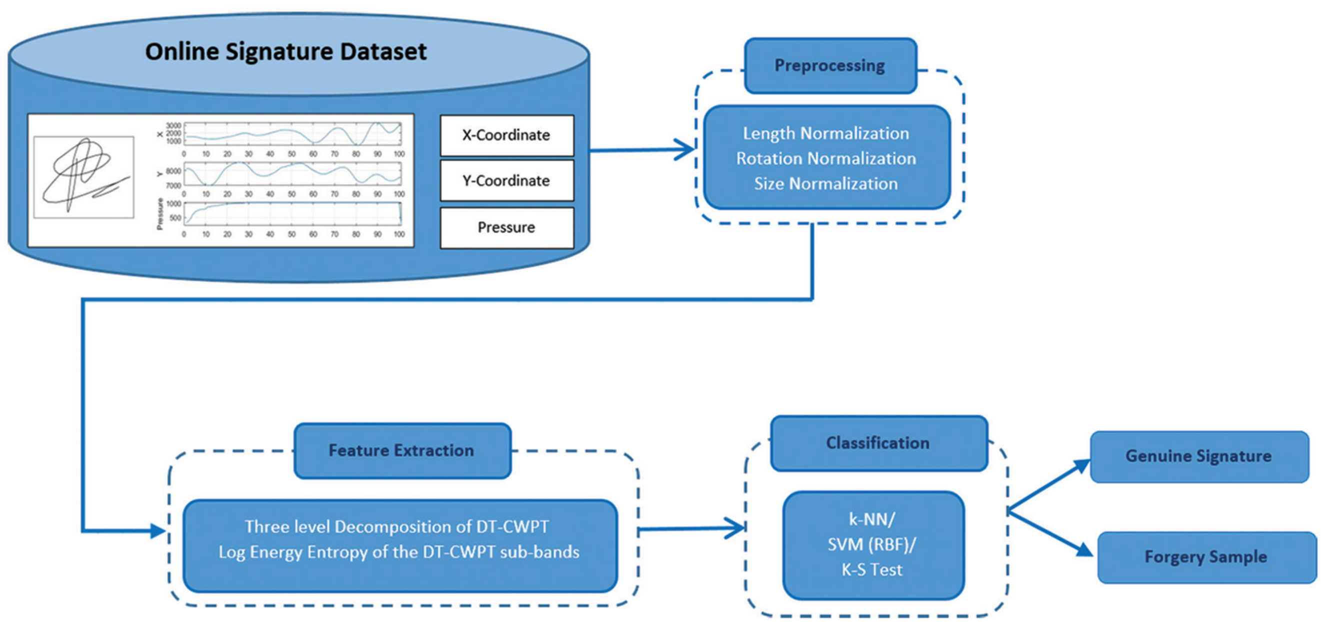

Figure 2: Block diagram of the presented online signature verification method

Table 1: The statistics of the used online signature datasets

Dataset	Users#	Per user		Dynamic information
		Genuine#	Forgeries#	
SVC2004	100	20	20 (skilled)	x-coordinate, y-coordinate, time stamp, button status, azimuth, altitude, pressure
MCYT-100	100	25	25 (skilled)	x-coordinate, y-coordinate, pressure, azimuth, altitude
NDSD	55	65	40 (skilled)	x-coordinate, y-coordinate, pressure, azimuth, altitude

NDSD – Noshirvani Dynamic Signature Dataset; MCYT – Ministerio De Ciencia Y Tecnologia

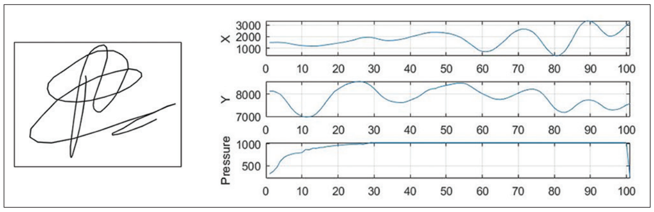

Figure 3: One Signature sample from online signature datasets Ministerio De Ciencia Y Tecnologia-100, with the corresponding dynamic information including x-coordinate, y-coordinate, and pressure

four other contributors for each contributor's signature. Dynamic information saved in Task 2 used in the presented method is shown in Table 1.

MCYT (http://atvs.ii.uam.es/atvs/mcyt100s.html) dataset composed of a fingerprint and an online Latin signature dataset.[38] Here, the public subset of MCYT contains signatures of 100 signers (MCYT-100) has been used. Dynamic information shown in Table 1 was saved using WACOM pen tablet, model INTUOS A6 USB with a capture area of size 127 × 97.[38] There are 25 genuine signatures and 25 skilled forgeries for each individual enrolled in the dataset. Forgeries were produced by five subsequent target users trying to copy the statistic images of the signature in 10 times.

NDSD (https://sites.google.com/site/meyahyatabar/ND SD % 20 Data.rar?attredirects = 0 and d = 1) is an online handwritten Persian signature dataset. WACOM INTUOS4 digitizing tablet was used to save dynamic information of signing process.[39] Signers signed on a plate of size 96 × 129 and the information was saved every 10 ms. These dynamic informations are shown in Table 1. Signers were in the range of 18–40 years old and signed 65 signatures in two different times with more than 3 days' interval. Further, 15 professional forgers forged 40 samples from 65 genuine signatures for each person.[39] As an example, one sample of online signature image from MCYT-100 signature dataset with the corresponding signals of horizontal position (x-coordinate), vertical position (y-coordinate), and pressure is shown in Figure 3.

Preprocessing

Among dynamic information describing the signing process available in the used datasets [Table 1], only three time signals including x-coordinate ($x(t)$), y-coordinate ($y(t)$), and pressure ($p(t)$) have been considered in the presented online signature verification method. Due to the intraclass variability, time signals corresponding to each signer have different length and size. Hence, some preprocessing tasks including length normalization, rotation normalization, and size normalization have been conducted on the mentioned time signals. Length normalization has been conducted to equalize the number of points of each signal. This is done using resampling of the signal. Further, in the process of gathering signatures, handwritten signatures are recorded in different angles, and so, rotation normalization is necessary to reduce intraclass variability. Rotation normalization has been conducted similar to.[40] In addition, since different dynamic ranges and to minimize this range and set the value changes in a specified interval, Minmax normalization has been conducted to set the values in [0,100] using Eq. 7, and then, they become standard using Eq. 8, as follows:[35]

$$f_{\text{Min-Max}}(t) = \left(\frac{f(t) - f_{Min}}{f_{Max} - f_{Min}}\right) \times 100 \tag{7}$$

$$f_s(t) = \frac{f_{\text{Min-Max}}(t) - \mu}{\sigma} \tag{8}$$

Where $f(t)$ is the values of a time signal and f_{Min} and f_{Max} are the minimum and maximum of the signal in hand, respectively. Notably, μ and σ are the average and standard deviation of the signal.

Proposed online signature verification method

After conducting the preprocessing tasks on the time signals including x-coordinate, y-coordinate, and pressure, feature extraction step has been conducted. Due to the intraclass and interclass variabilities, an effective signature verification system should be considered features with the ability to minimize intraclass variability and maximize interclass variability. The best way to do this is using the multiscale modeling.[21,29] In this article, a global feature extraction approach using DT-CWPT as multiscale modeling tool has been considered. To extract features from time signals, three-level decomposition of DT-CWPT has been performed on each time signal. The output subbands include the local characteristics of the original signal in three-level depth of decomposition. However, the number of these coefficients is so huge to directly use as the classifier input.[41] Therefore, some features should be derived from these coefficients to enhance the performance of the classifier.

To extract features, entropy concept has been used. Entropy is a common concept in many fields of research, mainly in signal processing,[41] and it is one of the widely used tools to extract features. Entropy measures the uncertainty of the information included in the signals.[41] There are several types of entropy including Shannon, Renyi, Tsallis, and log energy.[41] Shannon entropy of a time signal as a well-known entropy is a measure of its average uncertainty and computed as follows:

$$S(f) = \sum_i p(f_i) ln \frac{1}{p(f_i)} = -\sum_i p(f_i) lnp(f_i) \tag{9}$$

Where f_i is the value of the time signal $f(t)$ at time t_i. Shannon entropy has been used in the structure of the signature verification system, until today.[42,43] In this article, the log energy entropy (LEE) has been used as follows:

$$LEE(f) = \sum_i \ln\left(f_i^2\right)$$

To extract features, LEE for all subbands obtained from three-level decomposition of DT-CWPT has been computed. DT-CWPT is composed of two wavelet packets and each of them yields to 16 (=2^{3+1}) subbands. Hence, totally, 32 (=16 × 2) subbands have obtained with three-level decomposition of DT-CWPT. LEE of all of these subbands has been computed and concatenated to each other to make a feature vector corresponding to the original signature.

To classify the signatures in the feature space, three approaches have been tested for classification: k-nearest neighbor (k-NN), SVM, and Kolmogorov-Smirnov (K-S) test as a statistical classification approach. k-NN is a nonparametric classification technique, i.e., it does not make any assumptions on the underlying data distribution.[44] k-NN works based on feature similarity and its output is a class membership, a discrete value to predict a class by a majority vote of its neighbors, and being assigned to the class most common among its k-NNs (https://blog.usejournal.com/a-quick-introduction-to-k-nearest-neighbors-algorithm-62214cea29c7).[44]

SVM classification as a supervised learning algorithm is one of the most popular classifiers used in machine learning, pattern recognition, and signal processing.[45] The idea behind the SVM classification is finding a hyperplane with best separation between the data in the feature space.[46] In the case of nonlinear decision function, a nonlinear transformation is used to map the data from the input feature space into a high-dimensional feature space. According to the Cover's theorem and for nonlinear transformation and high-dimensional space, the patterns are linearly separable with high probability in the new feature space.[45] This nonlinear transformation is performed using kernel functions. There are different types of kernel including linear, polynomial, and radial basis function (RBF). RBF is one of the most popular kernel functions whose value depends on the distance from the origin or from some point. RBF kernel for two feature vectors X and X' is defined as follows:

$$K\left(X, X^{'}\right) = \exp\left(-\frac{X - X^{'2}}{2\sigma^2}\right) \tag{11}$$

Where ‖ ‖^2 is the squared Euclidean distance and σ is a free parameter. Equivalently, with $\gamma = \frac{1}{2\sigma^2}$, we have:

$$K\left(X, X^{'}\right) = \exp\left(-\gamma X - X^{'2}\right) \tag{12}$$

K-S test is a nonparametric hypothesis test which is widely used for comparing two given distance distributions of a single independent variable.[47] The K-S static for comparing two distributions P_0 and P is computed as follows:[47]

$$D_{mn} = \left(\frac{mn}{m+n}\right)^2 max_{-\infty<x<+\infty} \left|F_m\left(x\right) - G_n\left(x\right)\right| \tag{13}$$

Where $F_m(.)$ and $G_n\,(x)$ are empirical cumulative distribution functions (c.d.f.) of two given distance distributions P_0 and P, respectively. The P value for this test of hypothesis is computed as follows:

$$P = P\left(D \geq D_{mn} \mid H_0\right) \approx 1 - \mathrm{H}(D_{mn}) \tag{14}$$

Where $H(.)$ is the c.d.f. of K-S distribution with the following definition:[47]

$$H\left(t\right) = 1 - 2\sum_{i=1}^{\infty}\left(-1\right)^{i-1} e^{-2i^2 t} \tag{15}$$

K-S test for two samples tests the null hypothesis $H_0 : P_1 = P_2$, i.e., two samples come from the same distribution and accept the null hypothesis, if the computed P value is greater than a significance level (α), else the null hypothesis is rejected. It should be noted that in testing hypothesis, two types of error can occur: type I and type II.[47] Type I error occurs if the null hypothesis is rejected when it is true, and type II error occurs if the null hypothesis is accepted when it is false. The probability of committing a type I error in a decision rule is called the significance level (α). Significance level is usually considered as 5%, or 1%, and the null hypothesis is accepted with confidence level 95% (= 1 − α, for α = 5%).[47]

Experimental protocol and performance evaluation

In this section, details of the conducted experiments for the presented online signature verification in addition to the obtained experimental results have been described. At the end of this section, the presented method has been compared with literature.

Experimental protocol for the proposed online signature verification method

Three time signals x-coordinate, y-coordinate, and pressure have been considered as dynamic information corresponding to each signature in the dataset. Preprocessing tasks including length normalization, rotation normalization, and size normalization have been conducted on the mentioned time signals. Feature extraction has been conducted based on DT-CWPT on the preprocessed data. DT-CWPT has two separate wavelet packets. To get features from the different scales of the signals, three-level decomposition of DT-CWPT has been performed on the signals. Therefore, 16 subbands have been obtained for each signal at each wavelet packet. Notably, the length of these subbands is dependent to the length of the input signal which is different between datasets and also their users. The number of these coefficients is so huge to directly use as the classifier input, and therefore, LEE of each DT-CWPT subband has been computed as features. These features have been computed for 32 all subbands from both first and second wavelet packets of the three-level decomposition of a DT-CWPT for every signature in the dataset and concatenated in a vector as a feature vector of the signature. Therefore, for each time signal, 32 entropy measures have been computed as features per time signal. Totally, with considering three mentioned time signals, the final feature vector of length 96 (= 3 × 32) has been considered for each signature in the dataset. These feature vectors have been computed for every genuine signatures and skilled forgery samples in the dataset and then fed into the classification step.

Three classification techniques including k-NN, SVM, and K-S test have been considered to make decision for acceptance/rejection of the test signature. The parameters of these classification techniques have been empirically set as follows; k-NN with $k = 3$ neighbors and SVM with RBF kernel and regularization parameter $C = 1000$. Regularization parameter trades off the correct classification of training examples against maximization of the decision function's margin (https://www.mathworks.com/help/stats/fitcsvm.html). These values of the parameters work well for the problem; however, we noted that they could be optimized which is not explored in this work.

Here, an online handwritten signature verification with the WI approach has been presented. It should be noted that there are no skilled forgeries for each user enrolled to the system in real situations. Hence, to applicability of the presented system, training of the classifier only conducted with genuine signatures available in the dataset. To train the classifier and to avoid overfitting, feature vectors of genuine signatures have been divided into three sets: training, validation, and testing with 60%, 20%, and 20% of data, respectively. Then, the feature vectors of all of the forgery samples have been added into testing set and the final results have been reported using three-fold cross validation.

In the case of K-S test, making decision for acceptance/rejection of the query signature is formulated as a testing hypothesis, i.e., decision between two existed status: the query signature belongs to the person having claimed his/her identity (null hypothesis) or it is a forged sample (alternative hypothesis) (https://machinelearningmastery.com/statistical-hypothesis-tests/). In other words, K-S test is a decision between the following hypotheses:

$$H_0 : P = P_0 \; vs. H_1 : P \neq P_0 \tag{15}$$

Where P_0 is considered as the distribution of distances between training samples of true person, and P is considered as the distribution of distances between the query signature with all true saved samples of the person having claimed. Two distributions P_0 and P include pairwise distances between feature vectors of the signatures. These distances have been computed using Euclidean distance, in this article. Naturally, two distance distributions P_0 and P are slightly different. However, in signature verification process, the goal is answering to the question that whether existed difference is significant for failing the null hypothesis (rejection the query signature) or not. To do this, K-S test with $\alpha = 5\%$ has been considered in our experiments. The signature in question has been claimed as a genuine signature (accept the null hypothesis) or a skilled forgery sample (reject the null hypothesis) with confidence level of 95% (= $1 - \alpha$). Obtained experimental results are provided in the next section.

Table 2: Obtained signature verification results on the three benchmark datasets (%)

Dataset	k-NN	SVM (RBF)	K-S test
SVC2004 (Latin)	FRR=1.02	FRR=1.75	FRR=0
	FAR=3.81	FAR=3.5	FAR=2.5
	EER=8.40	EER=6.13	EER=1.8
MCYT-100 (Latin)	FRR=2.2	FRR=0.9	FRR=0
	FAR=0.9	FAR=0.5	FAR=0.4
	EER=4.0	EER=4.46	EER=0.2
NDSD (Persian)	FRR=10.92	FRR=9.85	FRR=0
	FAR=4.12	FAR=3.95	FAR=0.51
	EER=9.24	EER=6.15	EER=0.3

FRR – False rejection rate; FAR – False acceptance rate; EER – Equal error rate; k-NN – k-nearest neighbor; SVM – Support vector machine; RBF – Radial basis function; K-S – Kolmogorov-Smirnov; NDSD – Noshirvani Dynamic Signature Dataset; MCYT – Ministerio De Ciencia Y Tecnologia

Performance evaluation of the proposed online signature verification method

The performance of the presented signature verification method has been evaluated using three well-known criteria: false rejection rate (FRR), false acceptance rate (FAR), and equal error rate (EER). FRR is the fraction of the genuine signatures which have been falsely rejected, FAR is the fraction of skilled forgeries falsely accepted, and EER is the error rate when FRR = FAR. The presented signature verification method has been implemented using a system with Intel Core i7-7500U for CPU and 12 GB of RAM. Here, the MATLAB implementation of DT-CWPT has been used with three-level decomposition (https://ilkerbayram.github.io/dtcwpt/),[28] and the implementation of the presented method has been conducted using the programming language MATLAB R2017a (Natick, Massachusetts, USA).

Online signature verification results and comparison with literature

Several experiments have been conducted using three datasets of two scripts, including SVC2004 and MCYT-100, as two publicly available benchmark Latin datasets, and NDSD as a benchmark Persian dataset. Obtained experimental results are shown in Table 2 for three classification techniques: k-NN, SVM, and K-S test. As shown in Table 2, among three classification techniques, the best results in all three datasets have been obtained using K-S test for decision-making about genuine/forgery of the query signature. After K-S test, SVM with RBF kernel and k-NN classification obtained the best results, respectively.

To better intuition about how to separate the genuine signatures from forgery samples in the space of their feature vectors, t-distributed stochastic neighbor

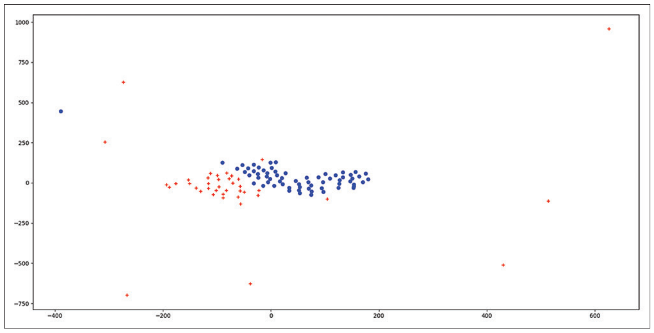

Figure 4: Separation of 65 genuine signatures from 40 forgery samples corresponding to one user from Noshirvani Dynamic Signature Dataset in the feature space, using t-distributed stochastic neighbor embedding algorithm. Genuine signatures have been shown using blue circle and the corresponding forgery samples have been shown with red "+" marker

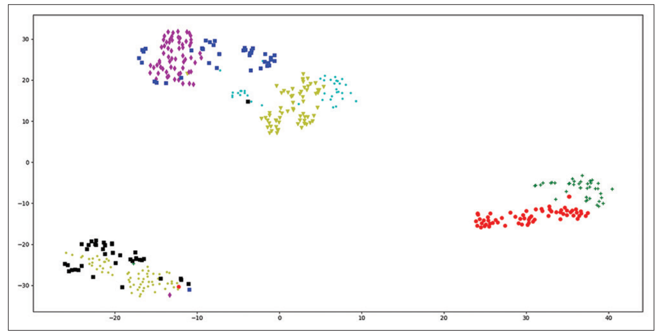

Figure 5: Separation of 65 genuine signatures from 40 forgery samples corresponding to four users from NDSD dataset in the feature space, using t-SNE algorithm. An obvious separation between four users is seen, in this figure. The pairwise markers (red 'o' and green '+'), (yellow '.' and black square), (violet '♦' and dark blue square), and (yellow '▼' and blue '.') have been used to show genuine signatures and forgery samples related to each user

Table 3: Comparison of the proposed online signature verification method with literature on three datasets: SVC2004, Ministerio De Ciencia Y Tecnologia-100, and Noshirvani Dynamic Signature Dataset

Dataset	Signature verification method	Type	FRR	FAR	EER
SVC2004	Wang *et al*.[29]	WD	-	-	6.65
	Alpar[30]	WD	1.67	3.33	3.41
	Chang *et al*.[31]	WD	-	-	4.87
	Nilchiyan and Yusof[32]	WD	3	3.5	3.5
	Rashidi *et al*.[40]	WD	-	-	3.37
	Nanni *et al*.[10]	WD	-	-	3.0
	Yang *et al*.[52]	WD	4.0	5.0	-
	Yahyatabar *et al*.[39]	WD	-	-	4.58
	Yahyatabar and Ghasemi[53]	WD	-	-	3.0
	Fayyaz *et al*.[49]	WI	-	-	0.83
	Proposed method	WI	0	2.5	1.8
MCYT-100	Nanni *et al*.[10]	WD	-	-	3.0
	Guru and Prakash[54]	WD	-	-	3.80
	Manjunatha *et al*.[55]	WD	3.83	0	1.92
	Diaz *et al*.[56]	WD	-	-	13.56
	Proposed method	WI	0	0.4	0.2
NDSD	Yahyatabar *et al*.[39]	WD	-	-	4.26
	Yahyatabar and Ghasemi[53]	WD	-	-	2.07
	Proposed method	WI	0	0.51	0.3

FRR – False rejection rate; FAR – False acceptance rate; EER – Equal error rate; WI – Writer-independent; WD – Writer-dependent; NDSD – Noshirvani Dynamic Signature Dataset; MCYT – Ministerio De Ciencia Y Tecnologia

embedding (t-SNE) (https://lvdmaaten.github.io/tsne/) algorithm has been used. t-SNE algorithm is a successful method which has been presented for dimensionality reduction introduced by van der Maaten and Hinton in 2008.[48] t-SNE technique is used for visualizing high-dimensional data into a low-dimensional space of two or three. Here, t-SNE algorithm has been used to visualize the feature vectors from feature space of 96-dimensional into 2D space. Figure 4 shows 2D projections of the feature vectors corresponding to the 65 genuine signatures and 40 skilled forgeries related to one user in NDSD dataset. Further, Figure 5 shows 65 genuine signatures and 40 skilled forgeries related to four different users in NDSD dataset. As shown in these Figures 4 and 5, the extracted features can make good separation between genuine signatures and skilled forgeries, which verifies the favorable signature verification results.

To compare the obtained experimental results with literature, several prominent online handwritten signature verification methods have been selected and compared using Table 3. Table 3 compares the presented signature verification method with some prominent methods implemented on three datasets: SVC2004, MCYT-100, and NDSD. As shown in Table 3, in the case of MCYTY-100 and NDSD datasets, the presented signature verification method outperformed the state-of-the-art. In the case of SVC2004, the state-of-the-art is dedicated to the presented method by Fayyaz *et al*.[49] It should be noted that Fayyaz *et al*.[49] used taught learning to learn feature from 17500 signature images of ATVS dataset,[50,51] and verification process has been conducted on SVC2004 to get 0.83% for EER. In comparison with this method, our presented method has lower computational cost which using hand-crafted features, instead of learning features.[49] However, Table 3 shows that in comparison with other prominent works, our method also obtained promising results in the case of SVC2004.

Proposed online signature recognition method

In this section, the proposed procedure for online signature recognition with the corresponding experiments, obtained results, and comparison with literature has been presented.

Proposed online signature recognition method

In this section, an online handwritten signature recognition has been presented. Similar to the presented signature verification method, three time signals x-coordinate, y-coordinate, and pressure have been considered and the preprocessing tasks have been conducted on them for each signature in dataset. Further, to extract features, a three-level decomposition of DT-CWPT has been performed on the mentioned three time signals. Then, entropy measures using LEE have been computed for each of the obtained subbands of DT-CWPT decomposition. These entropies have been concatenated to each other to make the feature vector related to each signature in dataset. Finally, to recognize the class of the query signature in the presented signature recognition method, two classifiers including k-NN and SVM have been considered.

Online signature recognition results and comparison with literature

Three benchmark online signature datasets, including SVC2004 and MCYT100, as two Latin datasets, and NDSD, as a Persian dataset, have been considered for experiments. Notably, in the case of signature recognition, only genuine signatures from each dataset have been used. The presented signature recognition method has been evaluated using true recognition rate, i.e., the percentage of truly classified signatures on the datasets. To classify the query signature, two classifiers with the following parameters have been used: k-NN with $k = 3$ and SVM with RBF kernel and the regularization parameter $C = 1000$. These parameters have been empirically set for our experiments. Notably, in the case of SVM classifier, in addition to RBF kernel, polynomial kernel and linear kernel have been used to evaluate the presented signature recognition method. However, the best results have been obtained by RBF kernel of SVM classifier. To avoid overfitting, final signature recognition results have been reported using three-fold cross validation.

To the best of our knowledge, K-S test has not been used for recognition purpose, until today. This also has not been used here. Authors believe that this is because of the high computations and inaccurate decisions that may occurred with K-S test in the recognition decisions. This is explained here. To recognize the class of the query signature, the comparison should be done with all classes of people registered in the system. Suppose that there exist M classes of writers. Therefore, computed pairwise distances between signatures in each class lead to make M distributions of distances. In addition, computed distances between the query signature and the signatures in each class make M other distributions. K-S static should be computed between distribution of one class as P_0, and distribution of the query signature and that class, as P_1. The computed K-S static is compared to the considered P value. Then, between all of the acceptable K-S statics, the class of the query signature has been selected as the class with the maximum K-S static. Therefore, as explained, the computations are increased and make a final decision can be confusing when some equal K-S statics have been computed.

The presented signature recognition method has been implemented using a system with Intel Core i7-7500U for CPU and 12 GB of RAM. Here, the MATLAB implementation of DT-CWPT has been used with three-level decomposition,[28] and the implementation of the presented method has been conducted using the programming language MATLAB R2017a.

Table 4 shows the obtained signature recognition results. As shown in Table 4, the best recognition rate has been obtained using SVM with RBF kernel on three considered datasets.

To visualize signature recognition results, t-SNE algorithm has been used to better intuition about how to separation of signatures from different classes in the space of the feature vectors. Here, t-SNE algorithm has been used to visualize the feature vectors from feature space of 96 dimensions into 2D space. As an example, eight users from NDSD dataset have been considered and 2D projections of the feature vectors corresponding to 30 signatures of each user have been shown using t-SNE algorithm, in Figure 6. As shown in Figure 6, there is a geed separation between genuine signatures from eight different classes in their feature vectors, which leads to obtain favorable online signature recognition results shown in Table 4.

To compare the signature recognition results with literature, some of the prominent presented signature recognition methods have been considered. It should be noted that, in comparison with literature on signature verification methods, fewer methods have been presented for online signature recognition.

Table 4: Obtained signature recognition results on the three benchmark datasets in terms of true recognition rate (%)

Dataset	k-NN	SVM (RBF)
SVC2004 (Latin)	76.56	99.84
MCYT-100 (Latin)	75.50	99.35
NDSD (Persian)	83.74	99.60

k-NN – k-nearest neighbor; SVM – Support vector machine; RBF – Radial basis function; NDSD – Noshirvani Dynamic Signature Dataset; MCYT – Ministerio De Ciencia Y Tecnologia

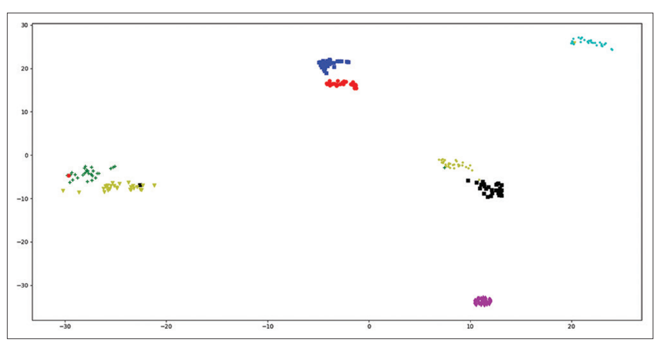

Figure 6: Separation of 240 signatures of 8 users (30 signatures per user), from Noshirvani Dynamic Signature Dataset in the feature space, using t-distributed stochastic neighbor embedding algorithm. Signatures from each class have been shown with different markers and discriminative colors

Table 5: Comparison of the proposed online signature recognition method with literature in terms of true recognition rate (%)

Method	SVC2004 (100 users)	MCYT-100 (100 users)	NDSD (55 users)	Private dataset
Shamsuddin and Mohamed[33]	-	-	-	80 (60 users)
Mohamed *et al.*[34]	-	-	-	90 (20 users)
Al-Mayyan *et al.*[35]	-	-	-	100 (108 users)
Fotak *et al.*[36]	-	-	-	94.25 (27 users)
Proposed method	99.84	99.35	99.60	-

NDSD – Noshirvani Dynamic Signature Dataset; MCYT – Ministerio De Ciencia Y Tecnologia

To the best of our knowledge, all of these methods have been evaluated on a private dataset and the comparison is not fair. However, their obtained true recognition rates and our obtained recognition rate are shown in Table 5. As shown in Table 5, the previous works presented good results and the method in the study by Al-Mayyan *et al.*[35] obtained the accurate recognition rate on its own created dataset composed of 2160 signature from 108 users. However, obtained signature recognition results on three benchmark dataset verify the suitably and effectiveness of the presented signature recognition method.

Conclusion and Future Works

In this paper, a novel approach for online handwritten signature verification and recognition has been presented. Dynamic information including three time signals x-coordinate, y-coordinate, and pressure have been considered to extract features. DT-CWPT has been used as a multiscale transform with three-level decomposition of the time signals. Using this decomposition, 32 subbands have been obtained for both of two wavelet packets of DT-CWPT. Then, instead of the huge number of coefficients of DT-CWPT subbands, entropy measures of each subband have been computed as feature corresponding to that subband. In the presented method, LEE has been used and the computed features of each subband have been concatenated to make the feature vector related to each signature in dataset. Three classifiers K-NN with $k = 3$, SVM with RBF kernel, and K-S test with confidence level of 95% have been used to final decision about the query signature to be a genuine signature or considered as a forgery sample. The performance of the method has been evaluated on three publicly available benchmark datasets: SVC2004 and MCYT-100 as two Latin datasets and NDSD as a Persian dataset. Experimental results show that the best classification has been conducted using K-S test on three mentioned datasets. Comparison with literature verifies that the presented method using DT-CWPT and LEE outperformed the state-of-the-art on MCYT-100 and NDSD datasets and also obtained really promising results on SVC2004 dataset.

Similar to the presented signature verification method, an online signature recognition method has been presented. After conducting the preprocessing step on three time signals: x-coordinate, y-coordinate, and pressure, their corresponding feature vectors have been computed using three-level decomposition of DT-CWPT and LEE measures. The classification for recognition of the user's class has been conducted using k-NN and SVM classifiers and the final reported results have been obtained by three-fold cross validation. Obtained results show favorable accuracy around 99% on three benchmark datasets which verifies the effectiveness of the presented method.

A possible future work for improving the performance of the presented online signature verification and recognition systems is using other multiscale transforms, other types of entropy measures, and other statistical test in the structure of the systems. In addition, the performance of DT-CWPT will be evaluated in the case of offline handwritten signature verification and recognition, by us.

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Conflicts of interest

There are no conflicts of interest.

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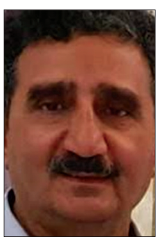

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